

# Disk galaxy rotation curves and dark matter distribution

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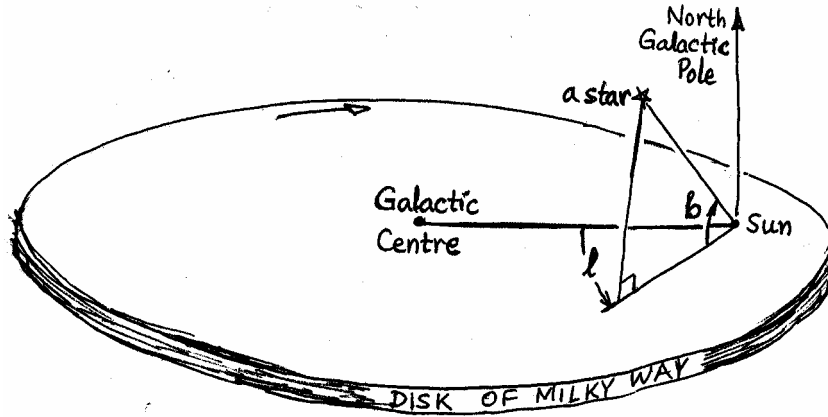
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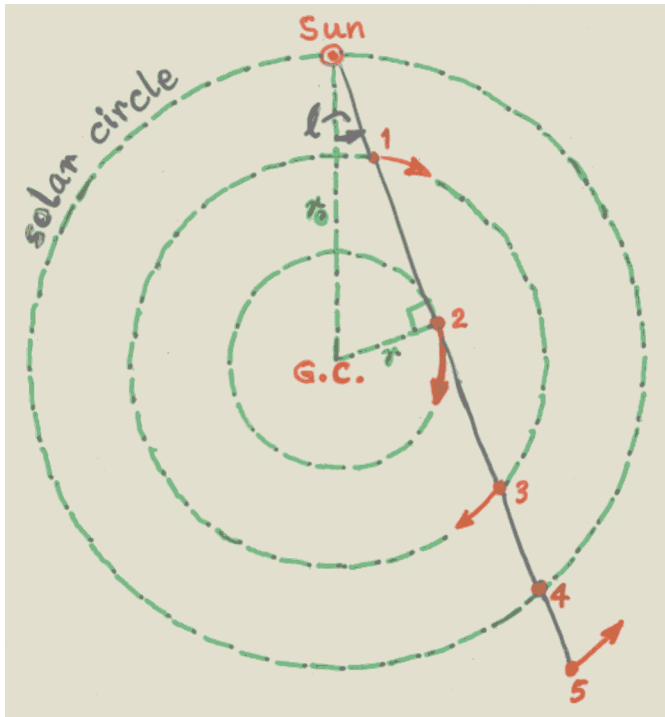
**Abstract.** After explaining the motivation for this article, we briefly recapitulate the methods used to determine the rotation curves of our Galaxy and other spiral galaxies in their outer parts, and the results of applying these methods. We then present the essential Newtonian theory of (disk) galaxy rotation curves. The next two sections present two numerical simulation schemes and brief results. Finally, attempts to apply Einsteinian general relativity to the dynamics are described. The article ends with a summary and prospects for further work in this area. Recent observations and models of the very inner central parts of galaxian rotation curves are omitted, as also attempts to apply modified Newtonian dynamics to the outer parts.

**Motivation.** Extensive radio observations determined the detailed rotation curve of our Milky Way Galaxy as well as other (spiral) disk galaxies to be flat much beyond their extent as seen in the optical band. Assuming a balance between the gravitational and centrifugal forces within Newtonian mechanics, the orbital speed  $V$  is expected to fall with the galactocentric distance  $r$  as  $V^2 = GM/r$  beyond the physical extent of the galaxy of mass  $M$ ,  $G$  being the gravitational constant. The run of  $V$  against  $r$ , for distances less than the physical extent, then leads to the distribution  $M(r)$  of mass within radius  $r$ . The observation  $V \approx \text{constant}$  for large enough  $r$ , upto the largest  $r$ , upto 100 kpc, thus shows that there is substantial amount of matter beyond even this largest distance. A spherically symmetric matter density  $\rho(r) \propto 1/r^2$ , characteristic of an isothermal sphere, leads to  $V = \text{constant}$ . Since this matter doesn't emit radiation, it is called dark matter. In general, the existence of dark matter is, by astrophysical definition, inferred solely from its gravitational effects. (Astro)particle physicists hope to change this by directly detecting dark matter particles. One often assumes an isothermal dark matter halo, although  $\rho(r) \propto 1/r^2$  is only one of many density profiles leading to  $V = \text{constant}$ , the others being disklike. For disklike mass distributions, in contrast to spherically symmetric ones, the circular speed at a given  $r$  is determined by matter distributed from 0 to  $r$  and also beyond  $r$ , as can be easily seen by applying Gauss' integral theorem (or law) to appropriately shaped closed volumes. Some textbooks make the error of integrating only upto  $r$ , leading to wrong results (Méra et al 2006).

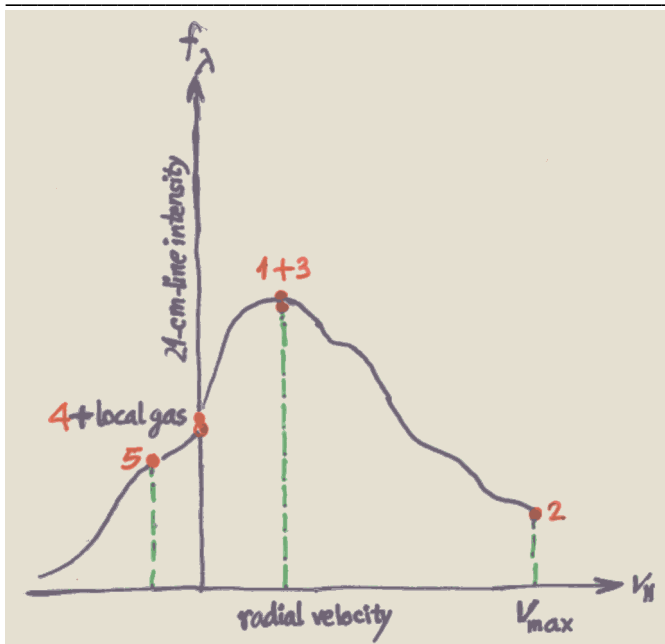


**Fig. 1** A schematic picture of Sun's location in Milky Way Galaxy, illustrating Galactic coordinates  $b$  (latitude) and  $\ell$  (longitude). Rotation is indicated by an arrow.

Recently, de Boer et al (2005) reanalyzed the public domain data from the 0.1 to 10 GeV all-sky  $\gamma$ -ray survey which was done by the satellite-borne telescope EGRET. They found that (1) excess diffuse  $\gamma$  rays of the same spectrum are observed in all the sky directions, and that (2) the spectral shape can be interpreted as annihilation of (dark matter) particles (and antiparticles) into intermediaries like  $\pi^0$  mesons and then to  $\gamma$ -ray photons, implying a mass of 60 to 70 MeV/ $c^2$  for the annihilating (dark matter weakly interacting massive) particle (or antiparticle). From the intensity variation of the diffuse  $\gamma$ -ray excess with respect to Galactic longitude and latitude (Fig. 1), and assuming a spherically symmetric component in the mass distribution, they derived an almost spherical isothermal profile plus substructure in the Galactic Plane in the form of two toroidal rings at 4.2 kpc and 14 kpc from the Galactic Centre. (The absolute normalization of the dark matter profile is tied to the local rotation speed 220 km/s at 8.3 kpc, Sun's distance from the Galactic Centre. This may need renormalization for consistency with better more recent determinations, e.g., like those of Xu et al (2006) and Hachisuka et al (2006).) The two rings produce, within Newtonian dynamics, the observed bumps in the detailed shape of the Galactic rotation curve. These rings are actually broken segments disposed around the Galactic Centre in ringlike structures, as also seen for OB stellar associations via their distribution and kinematics (Mel'nik 2006). In general, nonaxisymmetric structures like spiral arms and bars should also be taken into account (e.g., Rix & Zaritsky 1995). Dekel (2005) cautions to choose carefully test particles to measure rotation curves in general, giving an example where low stellar speeds turn out to be a red herring, detailed disks' merger model for the elliptical galaxies in question showing orbits consistent with dark matter halo *and* low stellar speeds. Aharonian et al (2006) describe the discovery of TeV  $\gamma$  rays from the Galactic Centre Ridge. However, in this review, we restrict ourselves to outer parts of galaxies, i.e., to sufficiently large  $r$ , where the rotation curve has stabilized to a flat shape on average.



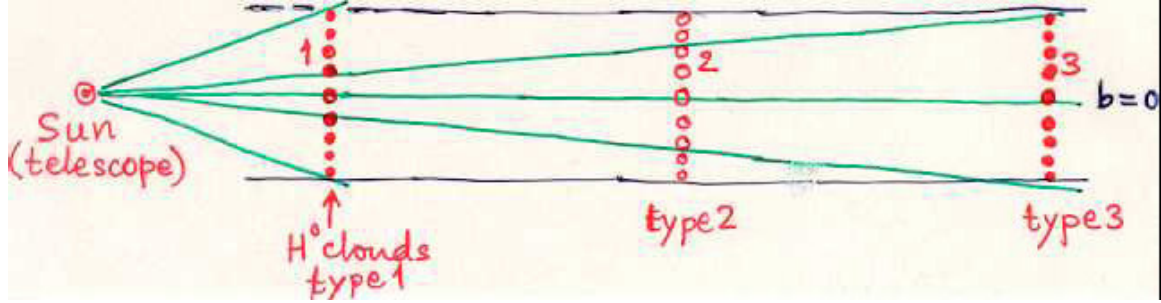
**Fig. 2a** Different types of clouds in a given direction. See Fig. 2b for the line profile.



**Fig. 2b** Neutral hydrogen line profile showing typical regions as numbered in Fig. 2a.

**Measurements in the Milky Way = Galaxy** (Shu (1982 or 1985), Binney & Tremaine (1987)). In the kinematic method of determining distances in the Galactic Plane in a given direction (see Fig. 2), the radial speeds of neutral hydrogen (i.e.,  $\text{H I} \equiv \text{H}^0$ ) clouds are measured by the Doppler shifts of the  $\lambda 21$  cm line corresponding to the electronic

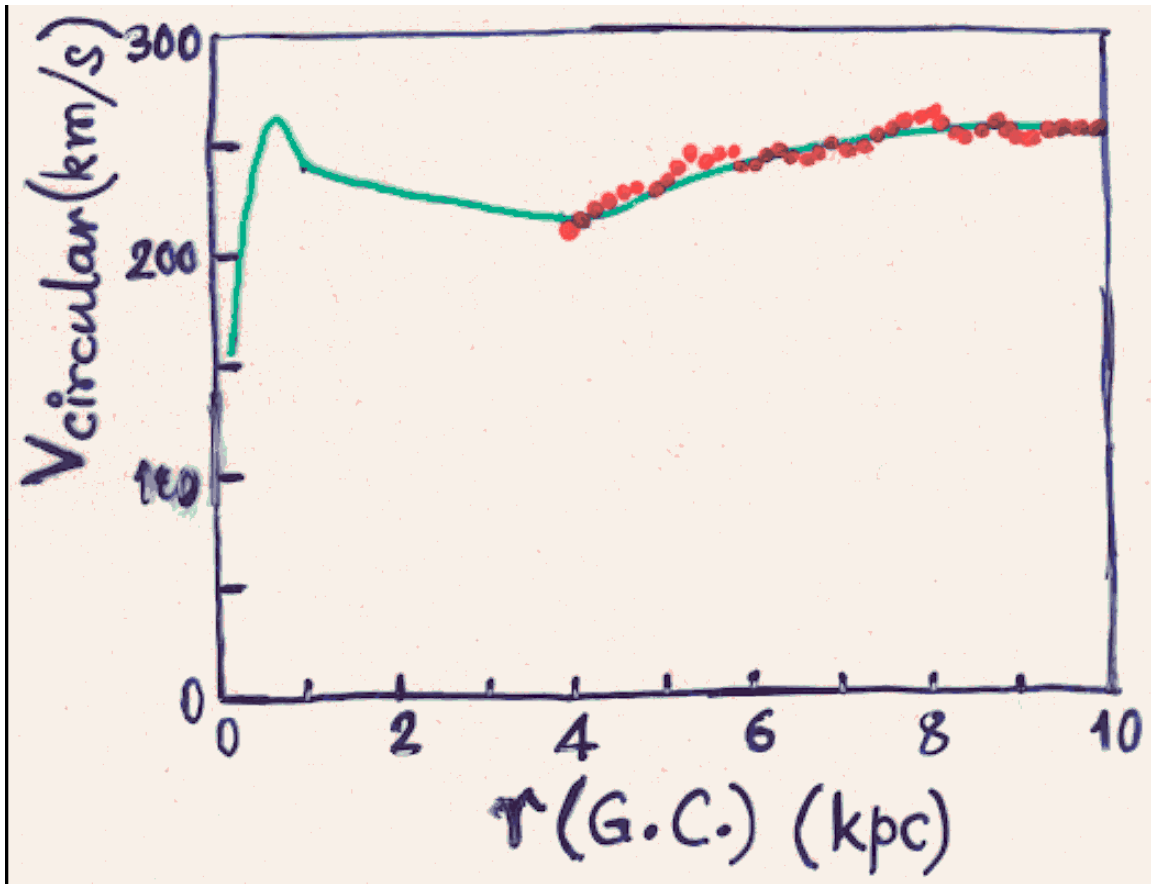
spin-flip transition of H-atom. The highest radial speed in the line-profile in Galactic longitude direction  $\ell$  gives the circular rotation speed  $V$  at distance  $r = r_{\text{sun}} \sin \ell$  from the Galactic Centre. The distance ambiguity for clouds of types 1 and 3 is removed (or resolved) by measuring their extent in Galactic latitude (see Fig. 3). Another way is to measure hydrogen absorption at radio frequencies, since distant sources show wider velocity range (Fish et al 2003). Such measurements give Milky Way rotation curve in its



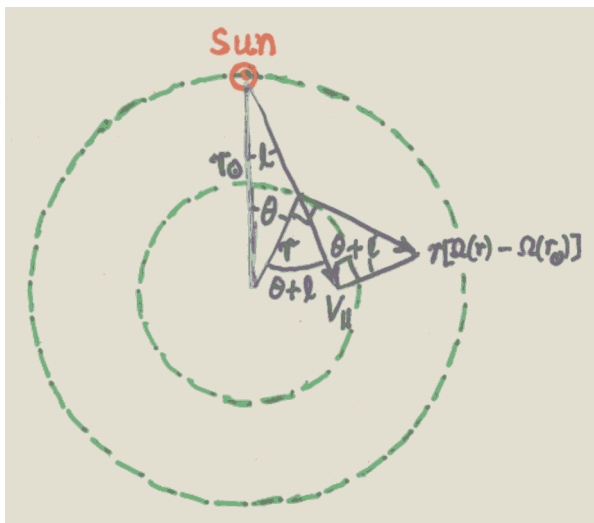
**Fig. 3** Resolving distance ambiguity by comparing widths in Galactic latitude.

outer parts, upto about the Solar Circle, as shown in Fig. 4. Relative to Sun (i.e., a frame of reference rotating with angular speed  $\Omega(r_{\text{sun}})$ ), the circular speed of a gas cloud at radius  $r$  is  $r[\Omega(r) - \Omega(r_{\text{sun}})]$  (see Fig. 5). The line of sight speed  $V_{\parallel}$  at a given Galactic latitude  $\ell$  is

$$V_{\parallel} = r[\Omega(r) - \Omega(r_{\text{sun}})] \sin(\theta + \ell) = r_{\text{sun}}[\Omega(r) - \Omega(r_{\text{sun}})] \sin \ell.$$



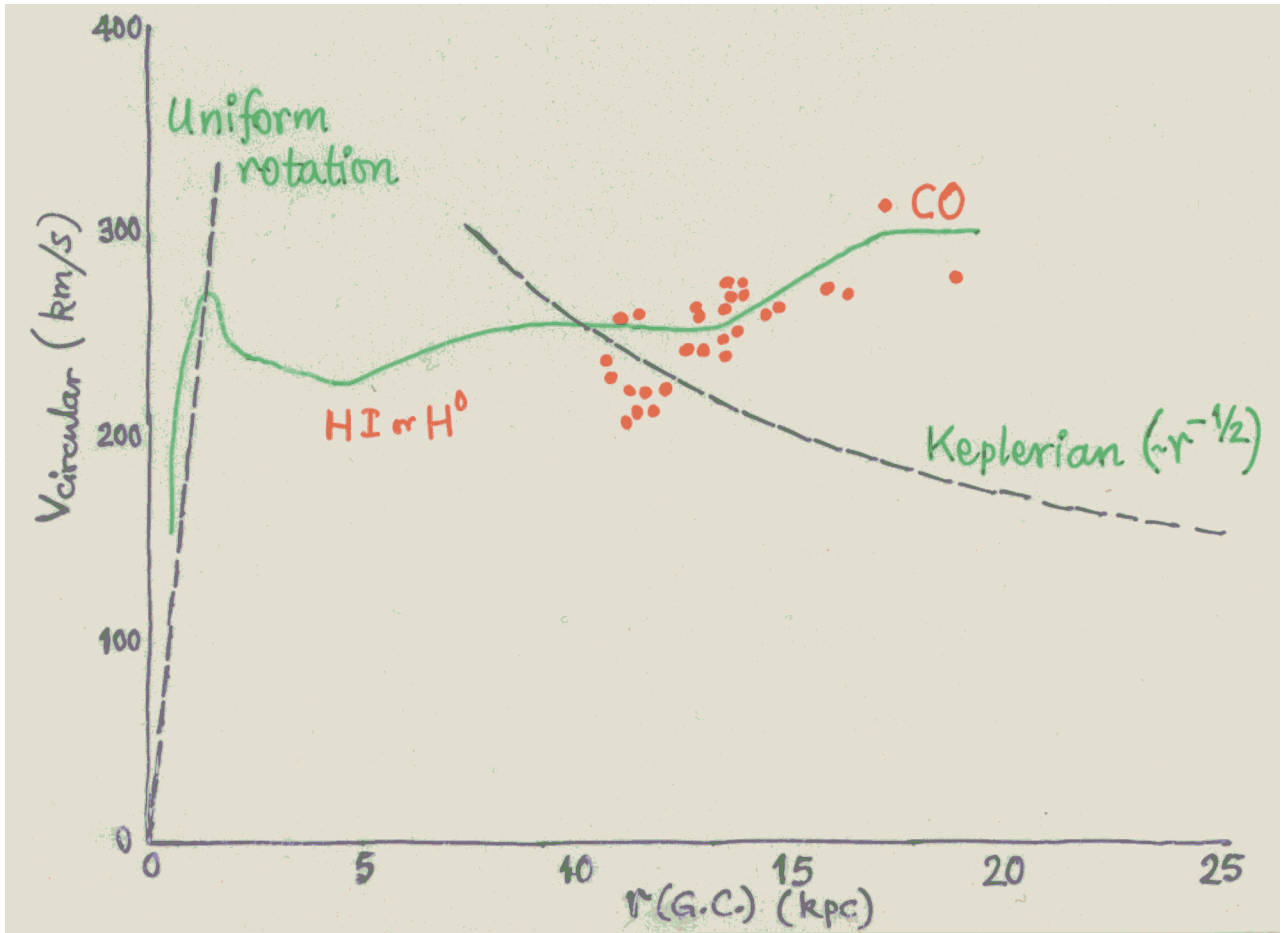
**Fig. 4** Milky Way rotation curve in outer parts



**Fig. 5** Geometry to calculate kinematic distances in the Galactic Plane.

So  $V(r) = r\Omega(r) = V_{\max} + r_{\text{sun}}\Omega(r_{\text{sun}})\sin\ell$ , where  $V_{\max}$  is the maximum value of  $V_{\parallel}$  from the line-profile. Beyond Solar Circle  $r = r_{\text{sun}}$ , giant H II  $\equiv$  H<sup>+</sup> complexes are used in place of H I ( $\equiv$  H<sup>0</sup>) regions. The tracers for these complexes are CO line (at  $\lambda \approx 1$  to 3 mm) and H109 $\alpha$  (at  $\lambda 6$  cm), among others. Fig. 6 shows Milky Way rotation curve beyond Solar Circle, with the “expected” curves for uniform rotation in the inner part and Keplerian behaviour in the outer part. The observations correspond roughly to uniform rotation in the inner part, although recently discovered fine structure there has other implications (e.g., Sofue & Rubin 2001 & references therein), which we omit from this review. In the outer part, the rotation speed is clearly super-Keplerian, on average flat, with some modulations, in this plot (Fig. 6) from Shu (1982). Binney & Tremaine (1987) summarize the detailed disk surface mass density model of Caldwell & Ostriker (1981), which is essentially a fit with 13 observationally determined parameters:

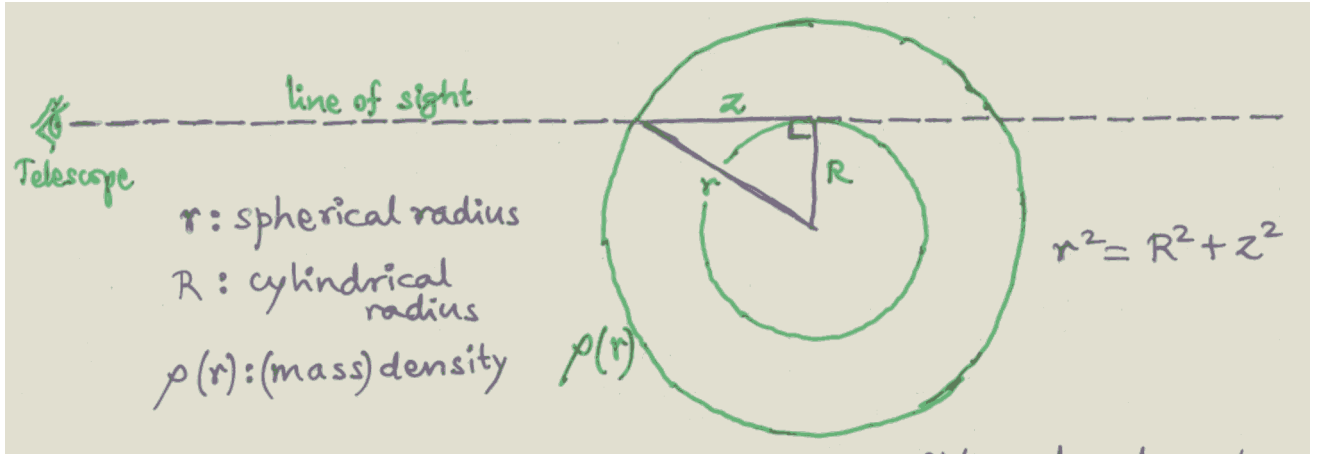
$\sigma(R) = \sigma_0[\exp(-R/R_1) - \exp(-R/R_2)]$ . The central model density is zero, which cannot accommodate a central mass concentration there, for which there is ample independent evidence (e.g., Sofue & Rubin 2001 & references therein). However, the model has toroidal rings at  $R_1$  and  $R_2$  for which there is separate observational evidence (de Boer et al 2005, Mel'nik 2006) as mentioned earlier. So a superposition of such a model with another component having centrally concentrated mass should fit the observed rotation curve for inner as well as outer parts of Milky Way Galaxy. For other detailed models of (especially the outer parts of) Milky Way rotation curve, see Cowsik et al (1996) and Dehnen & Binney (1998).



**Fig. 6** Milky Way rotation curve beyond the solar circle.

**Rotation curves of other galaxies.** Assuming Newtonian gravitation to predominantly determine the dynamics, the mass  $M$  may be estimated from orbital speed  $V$  in ellipticals as well as spirals. The mass  $M$  interior to  $r$  is roughly  $M(r) = rV^2/G$ , with  $r$  the deprojected distance and  $V$  the (spread in) random speeds for ellipticals, while in spirals it refers to the circular speed about the galaxian centre. For a disk galaxy it is more meaningful to take  $M(R)$  as the mass within a *cylinder* of radius  $R$ , while for a spheroidal galaxy  $M(r)$  is more conveniently the mass within a *sphere* of radius  $r$ . Observationally, measurements are possible only along the total line of sight through a galaxy (Fig. 7). So the cylindrical radius is more pertinent for observations. However, the practical difference between the two is not large for actual galaxies, as is seen from the following example. Take  $\rho(r) = C/r^2$ . Then the surface mass density is

$$\begin{aligned}\sigma(R) &= \int_{\text{all } z} dz \, \rho(r), \text{ where } r^2 = R^2 + z^2 \text{ (Fig. 7)} \\ &= \pi C/R.\end{aligned}$$



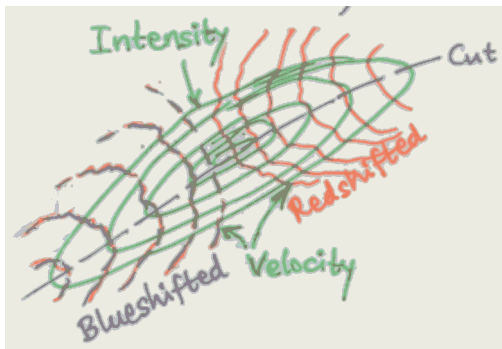
**Fig. 7** Line of sight through a galaxy, showing relation between spherical radius  $r$  and cylindrical radius  $R$ .

So the masses within a sphere of radius  $r$  and a cylinder of radius  $R$  are

$$M_{\text{sph}}(r) = \int_0^r dr 4\pi r^2 \rho(r) = 4\pi C r, \text{ and } M_{\text{cyl}}(R) = \int_0^R dR 2\pi R \sigma(R) = 2\pi^2 C R.$$

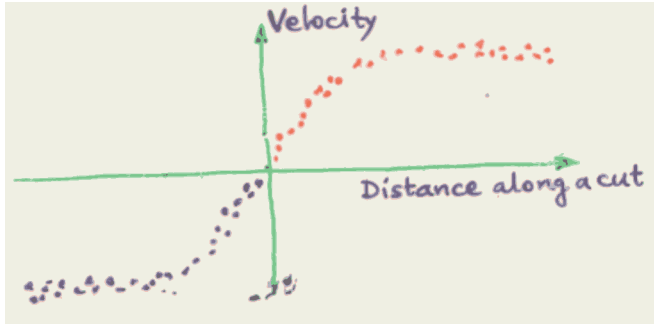
For  $r = R$ ,  $M_{\text{sph}}/M_{\text{cyl}} = 2/\pi \sim 1$ .

In general, (radio) line observations give data on the intensity and polarization of the targeted emitting matter in very many velocity (as implied by shifted frequency) channels in each pixel  $\equiv$  restoring beam. These data can be viewed and plotted in many different ways. The two most popular presentations give (1) a (polarized or total) intensity contour map with colour-coded iso-velocity contours superposed (Fig. 8), and (2) a cut through such a map showing a plot of velocity vs distance along the cut (Fig. 9).

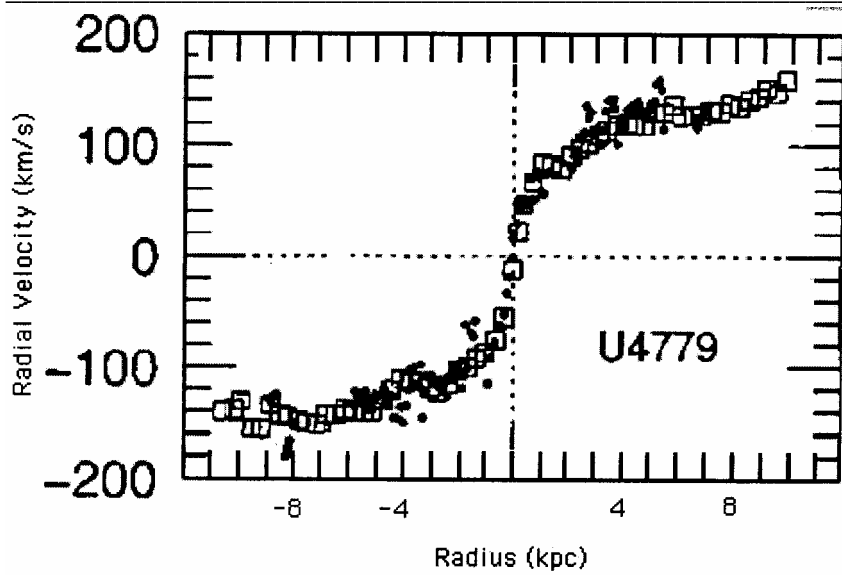


**Fig. 8** Schematic representation of iso-velocity contours superposed on intensity contour map (called *spider diagram*)

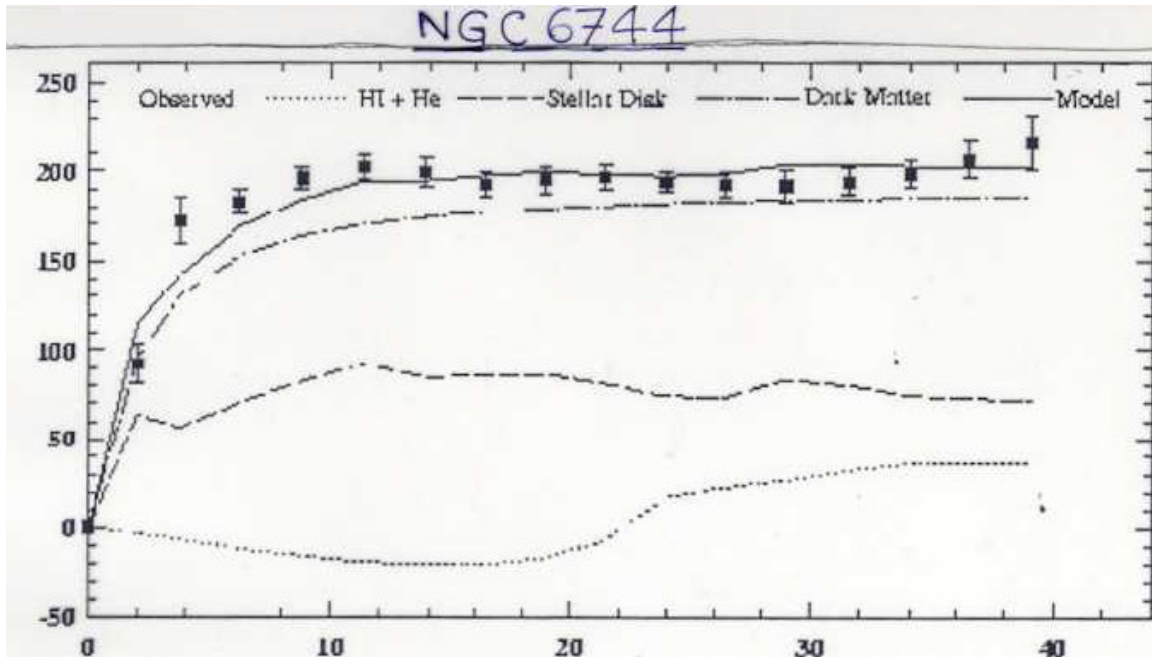




**Fig. 9a** Schematic galaxy rotation curve, such as may be derived from a spider diagram like in Fig. 8.



**Fig. 9b** An observed galaxy rotation curve (Courteau (1997)) for UGC4779 = NGC2742.



**Fig. 10** Rotation curve of NGC6744 to large galactocentric distance. (Velocity in km/s vs deprojected distance in kpc.) (From Ryder et al (1999).)

The velocity is always corrected for projection using an estimate of the inclination angle of the disk to the line of sight. There are other ways of getting the rotation curve, defined as such a position velocity plot (cf., e.g., Sofue & Rubin (2001)). A neutral hydrogen (i.e.,  $H^0$ ) study of NGC 6744 out to 40 kpc is shown in Fig. 10 (Ryder et al (1999)). UGC2885, NGC 5533 and NGC6674 rotation curves are measured to ~70 kpc (Sanders et al (1996)).

In the radio band, rotation curves can be measured out to about 2 to 3 Holmberg radii (Martín 1998b), while in the optical band, it is possible to measure only to about 0.5 times Holmberg radius, Holmberg radius corresponding to the isophote at a specific well-defined low surface brightness, about 1 to 2% above the background sky brightness (see, e.g., Binney & Tremaine (1987)). Other scales used to gauge the extent of disk galaxies are exponential disk scale length  $r_d$  from I-band photometry and the radius  $R_{opt}$  encompassing 83% of the total integrated light, fruitfully used by Catinella et al (2005), who construct template rotation curves by combining data on about 2200 disk galaxies, fitting for the amplitude  $V_0$ , exponential scale  $r_{pe}$  of the inner region and the slope  $\alpha$  of the outer part:  $V_{pe}(r) = V_0[1 - \exp(-r/r_{pe})](1 + \alpha r/r_{pe})$ ,  $pe$  standing for “polyex”, the name of the model, and  $r$  &  $r_{pe}$  expressed in units of  $r_d$  or  $R_{opt}$ . Detailed mapping of radial distribution of visible and dark matter in disk galaxies requires use of luminosity profiles and extended  $H^0$  rotation curves in addition to optical rotation curves. Martín (1998a,b) has collated  $H^0$  maps from literature published between 1953 to 1995 into a uniform catalogue of about 1400 disk galaxies, and analyzes some of the data in an attempt to derive features and relations common to most galaxies. It is sufficient to use angular measure (arcminutes or arcseconds) for the radial distances  $r$ ,  $r_{pe}$ ,  $r_d$  and  $R_{opt}$  for such

compilations. A translation to physical units (kpc) is needed only for going to mass models.

**(Newtonian) theory of (disk) galaxy rotation curves.** Binney & Tremaine (1987) give a detailed treatment of observations, models and theory of galaxian dynamics within Newtonian gravitational and dynamical framework. Saslaw (1985) treats gravitational systems in general from a much broader perspective, and, in the process, gives a concise summary of the essential Newtonian methodology. We refer the reader to these (and other possibly more recent) studies for details, and give here only a few glimpses, hopefully enough to whet the appetite for more.

Relation between  $\Phi(r)$  and  $\rho(r)$  [or  $V^2(r)$  and  $M(r)$ ]. The gravitational potential energy per unit mass is called *Newtonian gravitational potential*  $\Phi$ . This is only a convenient mathematical quantity and has no physical existence in Newtonian theory, which is a simultaneous far-action theory rather than (special) relativistically propagating field theory like (classical) electromagnetism. (Cf. Graneau & Graneau (1993) for an elaboration of this point. See also Banhatti & Banhatti (1996).)

For a test particle in a circular orbit at radius  $r$  in a spherically symmetric mass distribution  $\rho(r)$ , the *circular speed*  $V(r)$  is found from

$$V^2(r) = r d\Phi/dr = rF = GM(r)/r = (4\pi G/r) \int_0^r dr' r' \rho(r'),$$

in which  $F$  is the (radial) force / unit mass, and  $M(r)$  is the mass within a sphere of radius  $r$ . The *escape speed*  $V_e = (2|\Phi(r)|)^{1/2}$ , since the kinetic and potential energies are just balanced at this speed.

Potential  $\leftrightarrow$  density pairs (& other quantities)

\*Spherically symmetric. For illustration, some simple potentials are listed.

# Point mass  $M$ .  $\Phi(r) = -GM/r$ ,  $V(r) = (GM/r)^{1/2}$  &  $V_e(r) = (2GM/r)^{1/2}$ .

# Homogeneous sphere.  $M(r) = (4/3)\pi r^3 \rho$ , with  $\rho$  uniform (i.e., independent of  $r$ ) &  $V(r) = (4\pi G\rho/3)^{1/2} r$ , rising linearly with radius. The orbital period is  $T = 2\pi r/V = (3\pi/G\rho)^{1/2}$ , independent of radius  $r$ . A test mass released at  $r$  oscillates harmonically around  $r = 0$ , where it reaches after  $t_{\text{dyn}} = T/4 = (3\pi/16G\rho)^{1/2}$ , the *dynamical time* of a system of mean density  $\rho$ . For radial size  $a$ ,

$$\Phi(r) = \begin{cases} -2\pi G\rho(a^2 - r^2/3), & r \leq a \\ -4\pi G\rho a^3/3r, & r \geq a \end{cases}.$$

# Without giving details, other systems of interest are isochrone potential, modified Hubble profile and power-law density (Binney & Tremaine 1987).

\* Flattened systems. Plummer-Kuzmin, Toomre's n, logarithmic. For details, see Binney & Tremaine (1987).

Poisson's equation for thin disks. For an axisymmetric system with density  $\rho(R,z)$ ,

$$\partial^2 \Phi / \partial z^2 = 4\pi G \rho(R,z) + (1/R)(\partial/\partial R)(R F_R); F_R \equiv -\partial \Phi / \partial R \text{ being the radial force.}$$

Near  $z = 0$ , 1st term on the RHS  $\gg$  2<sup>nd</sup> term, so that  $\partial^2 \Phi / \partial z^2 = 4\pi G \rho(R,z)$ .

So Poisson's equation for a thin disk can be solved in two steps: (1) Using surface density (zero thickness), determine  $\Phi(R,0)$ . (2) At each radius  $R$ , solve this simplified Poisson's equation for the structure normal to the disk.

Disk potentials. By separating variables in cylindrical polar coordinates, surface density  $\sigma(R)$  and potential  $\Phi(R,z)$  are related by

$$\Phi(R,z) = -2\pi G \int_0^\infty dk \exp(-k|z|) J_0(kR) \int_0^\infty dR' R' \sigma(R') J_0(kR'),$$

where  $J_0$  is a Bessel function. Writing  $S(k) = -2\pi G \int_0^\infty dR R \sigma(R) J_0(kR)$ , the circular speed is given by

$$V^2(R) = R(\partial \Phi / \partial R)_{z=0} = -R \int_0^\infty dk k S(k) J_1(kR), \text{ using } dJ_0(x)/dx = -J_1(x).$$

\* Examples applying these formulae

# Rotation curve of Mestel's<sup>(22)</sup> (1963) disk:  $\sigma(R) = \sigma_0 R_0 / R$ .

Calculation gives uniform, i.e.,  $R$ -independent, circular speed:  $V^2 = 2\pi G \sigma_0 R_0$ .

Since  $M(R) = 2\pi \int_0^R dR' R' \sigma(R')$ , this can also be written  $V^2 = GM(R)/R$ , as for a spherical system, which is true only for Mestel's disk.

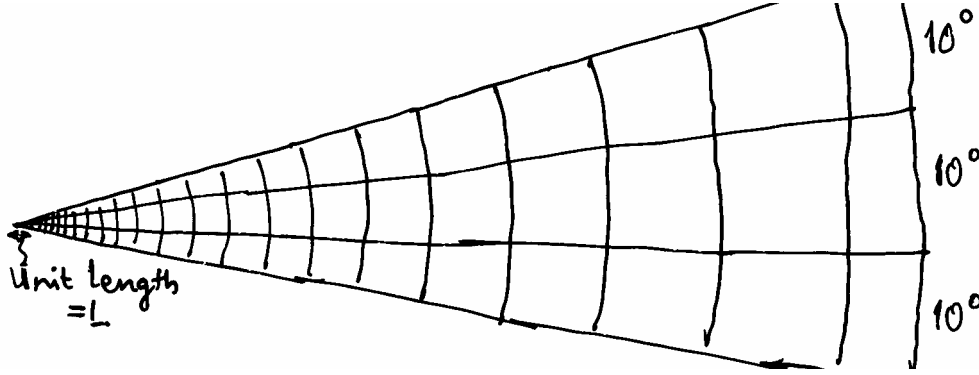
# Exponential disk:  $\sigma(R) = \sigma_0 \exp(-R/R_d)$ .

Calculation gives  $V^2(R) = 4\pi G \sigma_0 R_d^2 [I_0(y)K_0(y) - I_1(y)K_1(y)]$  where  $y \equiv R/2R_d$ , and  $I_0, K_0, I_1, K_1$  are Bessel functions.

Deducing  $\sigma(R)$  given  $V(R)$  is formally possible, but involves differentiating the noisy observed function  $V^2(R)$ , which numerically worsens the error and is unstable.

**Fourier (numerical) method in cylindrical polar coordinates** (Byrd et al 1986) With  $u \propto \ln R$ , a rectangular grid in  $(u, \phi)$  plane, where  $-\infty < u < \infty$  &  $0 \leq \phi$  (the azimuthal angle)  $\leq 2\pi$ , generates, in  $(R, \phi)$  plane, cells that become smaller as  $R \rightarrow 0$ . This is well-suited to (numerical simulations of) centrally concentrated disks. A particularly efficient implementation is R. H. Miller's (1970s) code, using "leapfrog" numerical scheme, extensively used at Turku (Finland) around 1986 and later (Valtonen et al (1990)) for

various aspects of disk galaxies including spiral arms, tidal interactions, Seyfert activity, etc. The N-body code uses 60000 particles in a smoothed potential. The particles are distributed on a 24 by 36 standard grid (part of which is shown in Fig.11), determined by  $r = L \exp(\lambda u)$ ;  $\lambda = 2\pi/36$  and  $u$  ranging from 0 to 23.5, so that  $r$  ranges from  $L$  to  $60.4L$ .



**Fig. 11** The two-dimensional polar coordinate grid used in R. H. Miller's numerical code.

The initial disk has

$$\sigma(R) = \begin{cases} [V_0^2/\pi G R] \cos^{-1}(R/A) & \text{for } R \leq A \\ 0 & \text{for } R > A \end{cases}, \text{ where } A \equiv \text{disk radius.}$$

Thus the circular speed  $V_0$  is, by integrating  $\sigma(R)$  to get the total galaxy mass  $M$ ,

$V_0^2 = \pi G M / 2A$ . The potential / unit mass is

$$\Phi(R) = \begin{cases} V_0^2 \ln(R/2A) & \text{for } R \leq A \\ -(2V_0^2/\pi) \sum_{\ell=0}^{\infty} [1 \cdot 3 \cdot 5 \cdots (2\ell+1) / \{2^\ell \ell! (2\ell+1)^3\}] (A/R)^{(2\ell+1)} & \text{for } R > A \end{cases}$$

The action of a spherical halo of density  $\propto 1/r^2$  is the same as if it was projected on the disk plane, as can be verified by calculation that the surface density of the projected halo has the same form as assumed. In fact, this is the motivation for using this form. *The halo acts on the disk, but is not acted on, either by itself, or by the disk. This is somewhat puzzling, and may give spurious effects.*

**Disk-halo break-up & a new calculation scheme** (Méra et al 2006). Poisson equation is numerically solved for an axially symmetric disk in cylindrical polar coordinates, using 250000 points = particles distributed along 500 rings of radii  $\propto i^2$ , where  $i$  is the ring number from centre outward, upto  $R_g$ , the radius of the finite disk. The force  $\mathbf{F} \propto \int [\sigma(R)/R^3] \mathbf{R} d\mathbf{R}$  acting on a given particle is discretized to

$$\mathbf{F}_i = \sum_{j \neq i} G(m_i m_j / d_{ij}^3) \mathbf{d}_{ij}, \text{ where } \mathbf{d}_{ij} = \mathbf{x}_j - \mathbf{x}_i.$$

also  $= m_i (v_i^2 / d_i) (\mathbf{x}_i / d_i)$  to give rotation curve,  $i = 0, 1, \dots, n$ .

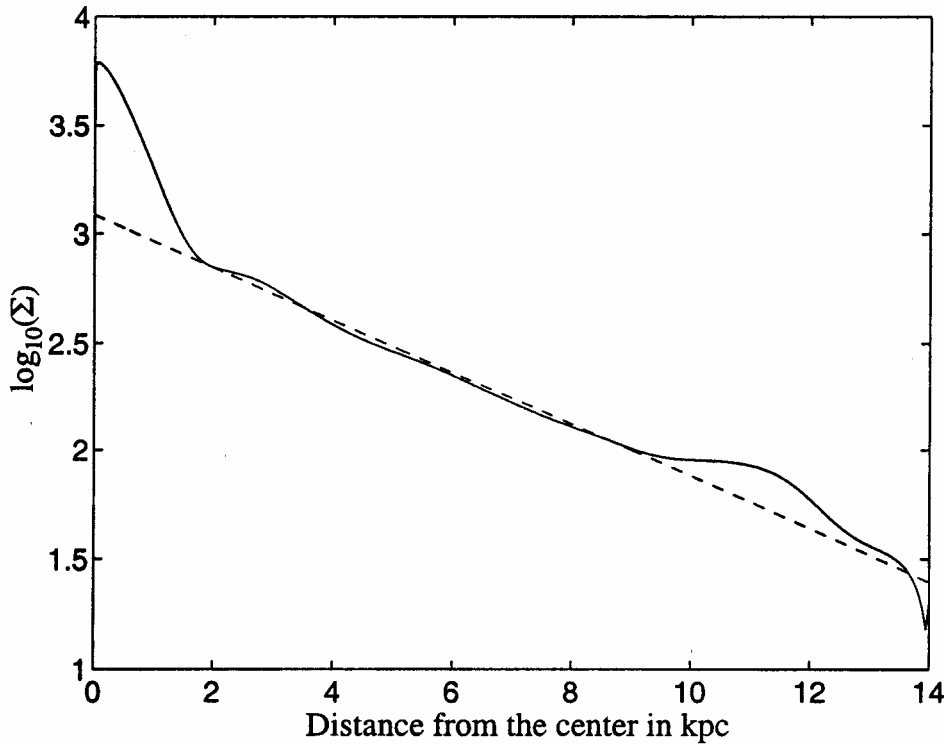
$d_{ij}^2 = d_i^2 + d_j^2 - 2d_id_j\cos(\theta_{ij})$ , so that, with  $F_{ij} = G(d_i - d_j\cos\theta_{ij})/d_{ij}^3$ , the set of equations

reduces to  $\sum_{j \neq i} m_j F_{ij} = v_i^2/d_i$ . This is a system of  $n$  linear equations, with  $n+1$  unknowns  $m_i$  (since we seek to invert  $V^2 \rightarrow \sigma$ ). The additional equation needed is provided by the total mass  $M_g$  of the galaxy:  $M_g = \sum_i m_i$ . Writing  $\mu_i = m_i/M_g \equiv \omega m_i$ , the  $n+1$  equations for  $n+1$  unknowns  $\mu_i$ , for each value of  $\omega$ , are (for  $i = 1, \dots, n$ ) ( $i = 0$  gives  $\mathbf{0} = \mathbf{0}$ ) :

$$\sum_{j=0}^n (j \neq i) m_j F_{ij} = \omega v_i^2/d_i, \quad \sum_{j=0}^n \mu_j = 1, \quad \text{with the constraint } \mu_i \geq 0.$$

The constraint restricts  $\omega$  between  $\omega_{\min}$  and  $\omega_{\max}$  which are very close to each other (within  $10^{-2}$  or less). The physical significance of the existence of the free parameter  $\omega$  is that the rotation curve is known only upto  $R_g$ , the radius of the finite disk. The range allowed for  $\omega$  corresponds to all possible extensions of the rotation curve beyond  $R_g$ . Since the range of  $\omega$  is very narrow, the mass of the galaxy from the known rotation curve is naturally found.

The method has been tested successfully for exponential disk, point mass and Mestel's disk. Fig.12 shows the surface density  $\sigma(R)$  for Milky Way derived in this way from the rotation curve of Vallée (1994).  $R_g = 14$  kpc is used from Robin et al (1992). Comparison with other results and taking into account MACHO gravitational lensing candidates toward LMC, a halo of the same radius as the disk is needed.



**Fig. 12** Surface mass density for Milky Way Galaxy compared with Méra et al's (2006) model (dotted)

### **General relativity vs. Newtonian gravity & dynamics**

Newtonian gravity & dynamics together form a simultaneous far-action theory. Newtonian gravitational potential (energy) is essentially a force field, giving the force / unit mass in space. This is true despite application of sophisticated mathematical techniques, as there is no (special) relativistic field propagation in Newtonian theory, which is not Lorentz-invariant, but Galilean, with absolute simultaneity, and space & time independent of each other. General relativity, on the other hand, is locally Lorentz-invariant, and is a genuine field theory with relativistic field propagation built into its structure.

*Galaxy rotation curves.* A galaxy is modeled as a stationary axially symmetric pressure-free fluid in general relativity. The rotation curve is derived by tracing paths of test particles, i.e., determining geodesics. Recent claim by Cooperstock & Tieu (2005) that such a procedure leads to a good fit to observed galaxy rotation curves, without having to assume existence of exotic dark matter, has been shown to be misguided by Vogt & Letelier (2005b, 2006), who infer that matter of negative energy density is implied in the disk. Korzyński (2005) shows that a singular disk is implied. Cross (2006) finds that correcting an internal inconsistency no longer leads to a flat rotation curve. Fuchs & Phelps (2006) point out the inadequacy of the model to reproduce local mass density and vertical density profile of the Milky Way. However, the idea of treating the nonlinear galactic dynamical problem using general relativity rather than Newtonian gravity, due to the inherent nonlinearity of self-gravity and (general relativistic) dragging of inertial frames in the rotating model spacetimes, is well worth pursuing further. An example is Vogt & Letelier (2005a). Another is Balasin & Grumiller (2006), who point out that the Newtonian approximation breaks down globally, even though it is valid locally everywhere, confirming Fuchs & Phelps' (2006) failed attempt to apply the model to the Milky Way Galaxy. Letelier (2006) summarizes the position, also referring to an earlier relevant study by González & Letelier (2000). Finally, it is worth mentioning Gödel's study of rotating spacetimes, examined from a broader perspective by Yourgrau (2005). See also Banhatti (2006).

### **Summary / Conclusion**

- # Galaxy rotation curves are flat to the greatest extent they can be measured. Precise observations have delineated the undulations in this overall flat structure, especially for Milky Way Galaxy.
- # There is a direct relation between rotation curves and mass models for galaxies. Dark matter is needed to fit the observations. In particular, a break-up of the mass distribution into a disk + a halo around the disk, is needed, for both luminous and dark matter components.
- # In addition to analytic approaches, numerical calculations / simulations in 2D polar coordinates are fruitful in relating mass models to observations.
- # It is not clear if general relativistic treatment gives anything more than Newtonian dynamics, but is worth exploring further.

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